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LETTER TO THE EDITOR

Comments on "Three-dimensional Thermal Analysis of Laminated Composite Plates" *Int. J. Solids Structures*, Vol. 32, pp. 593–608 (1995)

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In the course of reproducing the solutions of 3-dimensional thermoelastic laminated plates of M. Savoia and J. N. Reddy's paper published in *Int. J. Solids and Structures*, Vol. 32, No. 5, pp. 593–608, we found errors in eqns (12a) and (17) in the paper. Drs Savoia and Reddy missed the transverse normal effects in the discretized equilibrium equations. The eqns (12a) and (17) are inconsistent with equilibrium eqn (7) where they kept the transverse normal term. Thus, eqn (12a) for cross-ply laminates should be corrected as follows.

$$-(C_{\alpha\alpha\alpha\alpha}\delta_\alpha^2 + C_{\alpha\beta\alpha\beta}\delta_\beta^2)U_\alpha + C_{\alpha3\alpha3}U_{\alpha,33} - (C_{\alpha\alpha\beta\beta} + C_{\alpha\beta\alpha\beta})\delta_\alpha\delta_\beta U_\beta + (C_{\alpha\alpha33} + C_{\alpha3\alpha3})\delta_\alpha U_{3,3} = B^\alpha(x_3) + (C_{\alpha\alpha\alpha\alpha}\alpha_{\alpha\alpha} + C_{\alpha\alpha\beta\beta}\alpha_{\beta\beta} + \underline{C_{\alpha\alpha33}\alpha_{33}})\delta_\alpha T(x_3) \quad (1)$$

The first of eqn (17) for antisymmetric angle-ply laminates should be corrected as follows.

$$-A_{1\alpha}U_\alpha^1 + A_{2\alpha}U_{\alpha,33}^1 - A_{3\alpha}U_\beta^1 + A_{4\alpha}U_{3,3}^2 - A_{5\alpha}U_\alpha^2 - A_{6\alpha}U_\beta^2 - A_{7\alpha}U_{\beta,33}^2 - A_{8\alpha}U_{3,3}^1 = (C_{\alpha\alpha\alpha\alpha}\alpha_{\alpha\alpha} + C_{\alpha\alpha\beta\beta}\alpha_{\beta\beta} + \underline{C_{\alpha\alpha33}\alpha_{33}} + 2C_{\alpha\alpha\alpha\beta}\alpha_{\alpha\beta})\delta_\alpha T(x_3). \quad (2)$$

The second of eqn (17) should be corrected as follows.

$$-A_{1\alpha}U_\alpha^2 + A_{2\alpha}U_{\alpha,33}^2 - A_{3\alpha}U_\beta^2 - A_{4\alpha}U_{3,3}^1 - A_{5\alpha}U_\alpha^1 - A_{6\alpha}U_\beta^1 + A_{7\alpha}U_{\beta,33}^1 + A_{8\alpha}U_{3,3}^2 = (C_{\alpha\alpha\alpha\beta}\alpha_{\alpha\alpha} + C_{\alpha\beta\beta\beta}\alpha_{\beta\beta} + \underline{C_{\alpha\beta33}\alpha_{33}} + 2C_{\alpha\beta\alpha\beta}\alpha_{\alpha\beta})\delta_\beta T(x_3) \quad (3)$$

where the underlined parts of eqns (1–3) are terms missing in the original paper.

The derivations following eqns (12a) and (17) also do not contain transverse normal terms, resulting in erroneous stress and deformation distributions through the thickness of the plates. We do not mention here the minor typo-errors and sign mistakes they made.

The transverse normal effects are not usually considered in the analysis of global behavior of the laminated composite plates under mechanical loading because their contributions are insignificant. However, they cannot be neglected in thermal problems in the prediction of interlaminar normal stresses through the thickness.

Furthermore, in the exact three dimensional analysis, they can never be ignored. We do not believe that the authors intentionally neglected the transverse normal terms. However, since this omission generated erroneous solutions, we prepared this Comments.

For verification, our corrected solutions are compared with those of Noor *et al.* (1994) who used a different approach from Savoia and Reddy's (1995). As shown in Fig. 1, our corrected solutions are the same as those of Noor's (1994).

Based on this comparison with Noor's result given as Fig. 1, it is evident that the ignorance of the transverse normal effects generates nontrivial errors. The error in the solutions of two problems which are steady state response of thermoelastic bending of sandwich square plate and transient response of cross-ply laminate under temperature changes are shown in Figs 2 and 3. The material data and the non-dimensional parameters are given as follows.

Material I

$$E_L = 200\,\text{Gpa},\quad E_T = 8\,\text{Gpa},\quad G_{LT} = 5\,\text{Gpa},\quad G_{TT} = 2.2\,\text{Gpa}$$

$$v_{LT} = 0.25,\quad v_{TT} = 0.35,\quad \alpha_L = -2\times10^{-6}/^\circ\text{K},\quad \alpha_T = 50\times10^{-6}/^\circ\text{K}$$

$$k_L = 50\,\text{W}/(\text{K}\cdot\text{m}),\quad k_T = 0.5\,\text{W}/(\text{K}\cdot\text{m})$$

Material II

$$E_I = 1\,\text{Gpa},\quad E_T = 2\,\text{Gpa},\quad G_{IT} = 0.8\,\text{Gpa},\quad G_{II} = 3.7\,\text{Gpa}$$

$$v_{IT} = 0.25,\quad v_{II} = 0.35,\quad \alpha_I = \alpha_T = 30\times10^{-6}/\text{K},\quad k_I = k_T = 50\,\text{W}/(\text{K}\cdot\text{m})$$

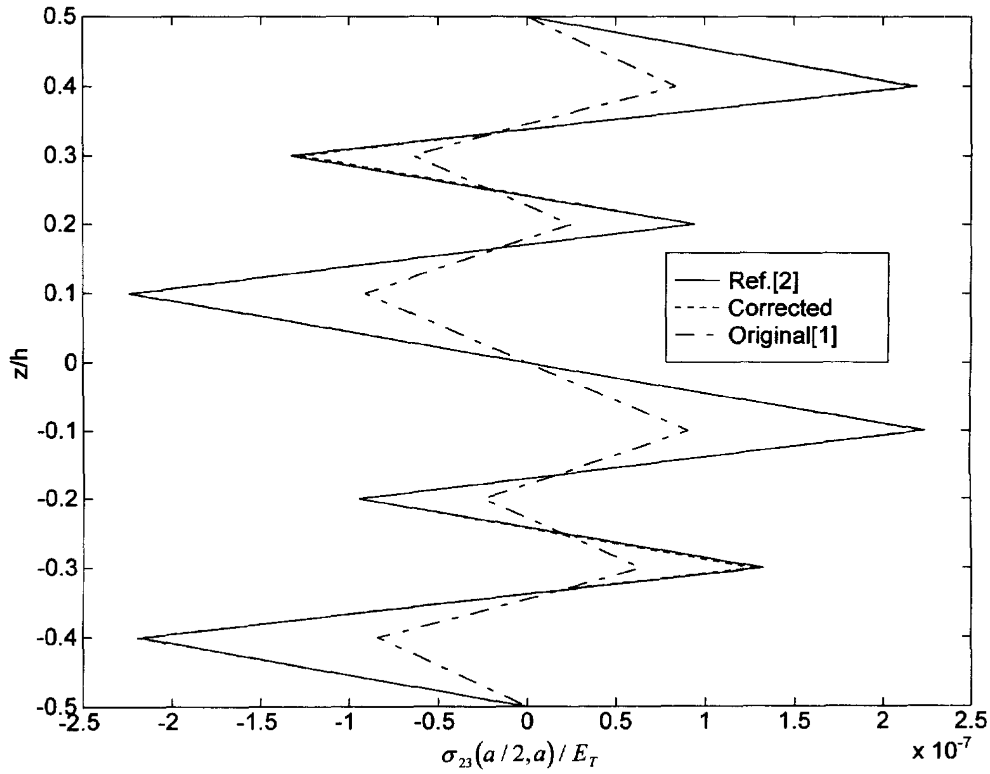
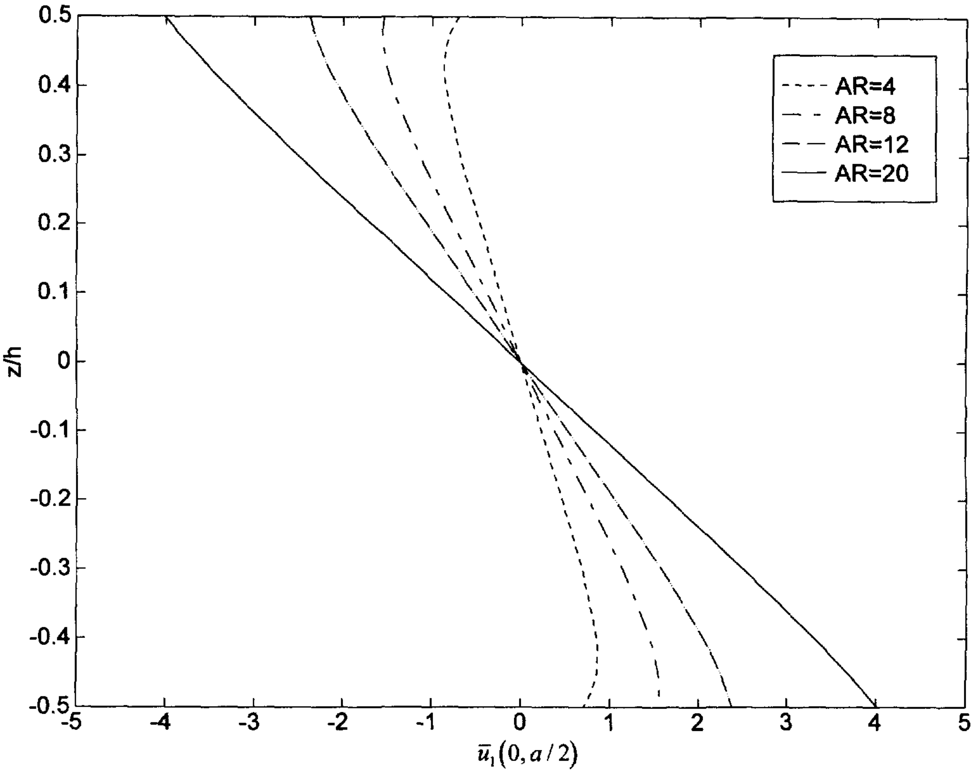
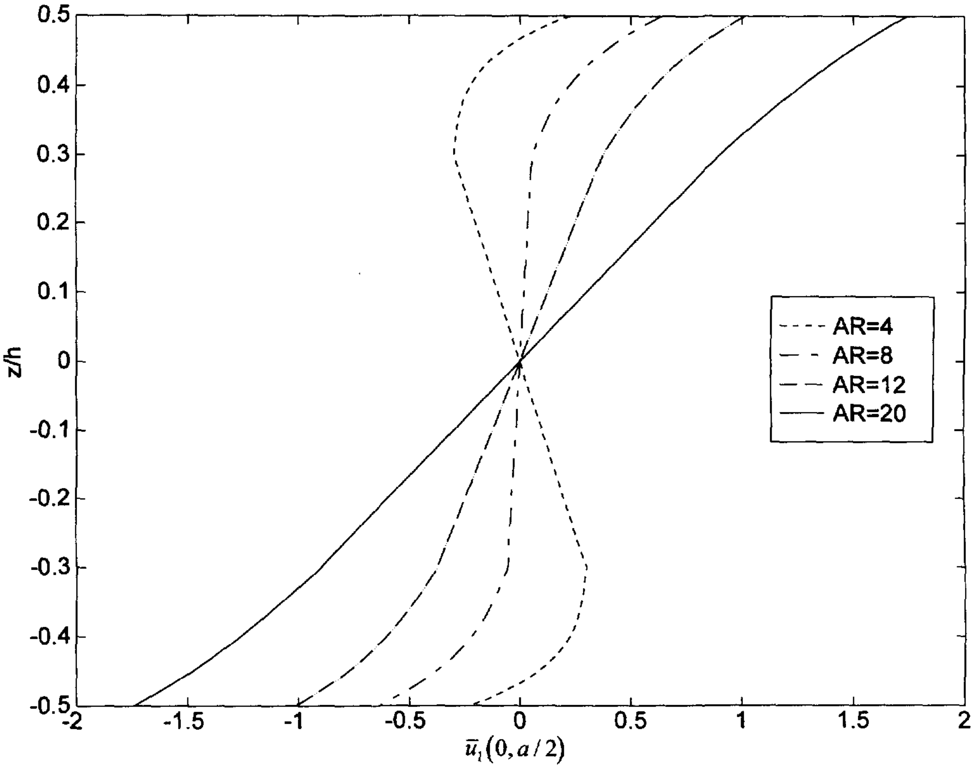


Fig. 1. Steady-state thermoelastic bending of a 10-layer cross-ply square plate $a \times a$ subject to a sinusoidal temperature rise at the two external faces ($m_1 = m_2 = 1$, $T^b_{11} = +\bar{T}$, $T^t_{11} = \bar{T}$). Through-the-thickness shear stress distributions for aspect ratio 10. $E_L/E_T = 15$, $G_{LT}/E_T = 0.5$, $G_{TT}/E_T = 0.3378$, $v_{LT} = 0.3$, $v_{TT} = 0.48$, $\alpha_L = 0.139\times10^{-6}$, $\alpha_T = 9\times10^{-6}$, $\bar{T} = 1$; $(0^\circ, 90^\circ, 0^\circ, 90^\circ, 0^\circ)_s$

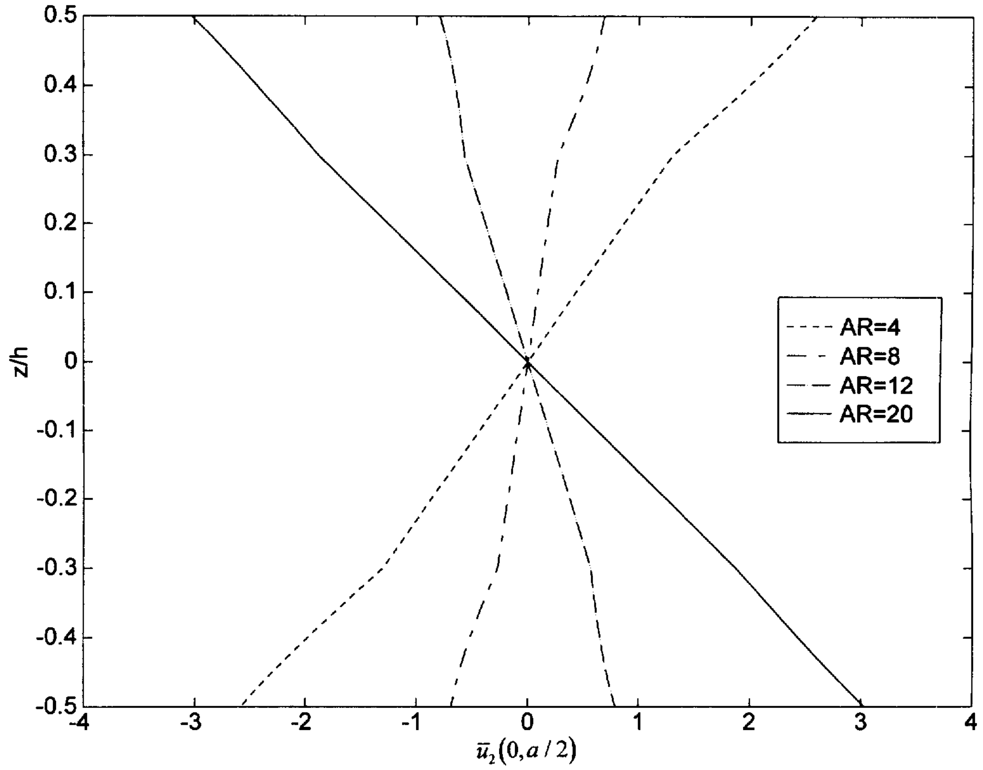


(a)

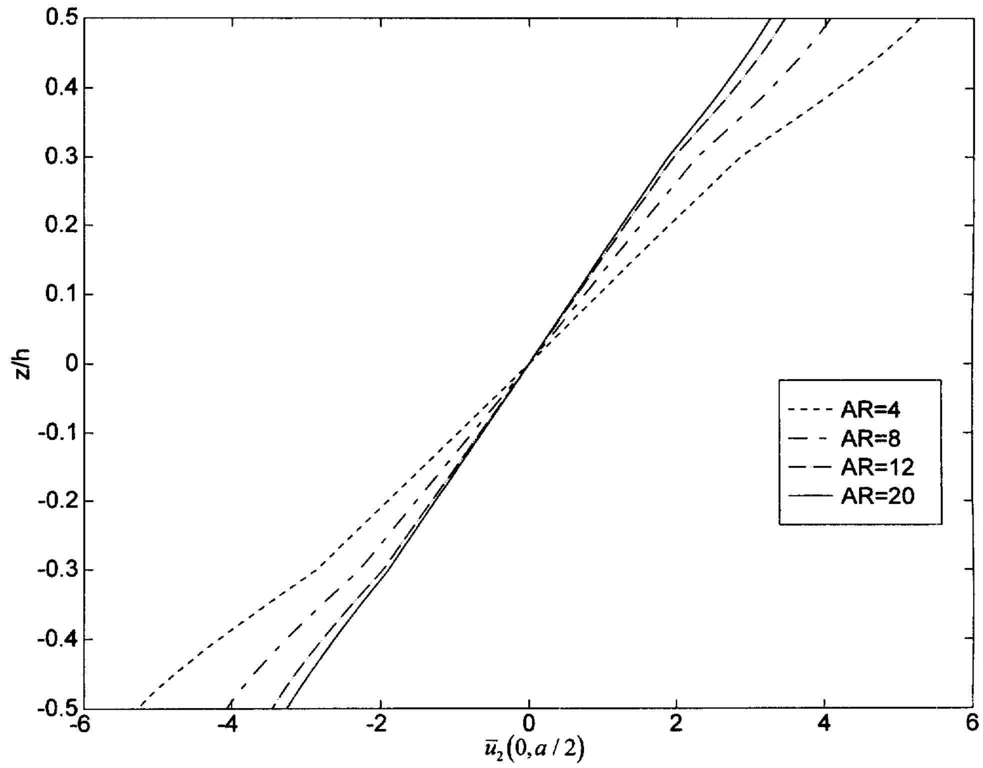


(a′)

Fig. 2. Steady-state thermoelastic bending of a sandwich square plate $a \times a$ subject to a sinusoidal temperature rise at the two external faces ($m_1 = m_2 = 1$, $T^{b}_{11} = +\bar{T}$, $T^{1}_{11} = -\bar{T}$). Through-the-thickness displacement and stress distributions for different values of the aspect ratio. Non-dimensional in-plane displacements (a, a', b, b') $\bar{u}_1$ and $\bar{u}_2$: Non-dimensional stresses $(c, c', d, d', e, e', f, f', g, g', h, h')$ $\bar{\sigma}_{11}$, $\bar{\sigma}_{22}$, $\bar{\sigma}_{33}$, $\bar{\sigma}_{21}$, $\bar{\sigma}_{31}$, $\bar{\sigma}_{12}$; Material I/II/I (0°/0°/0°)

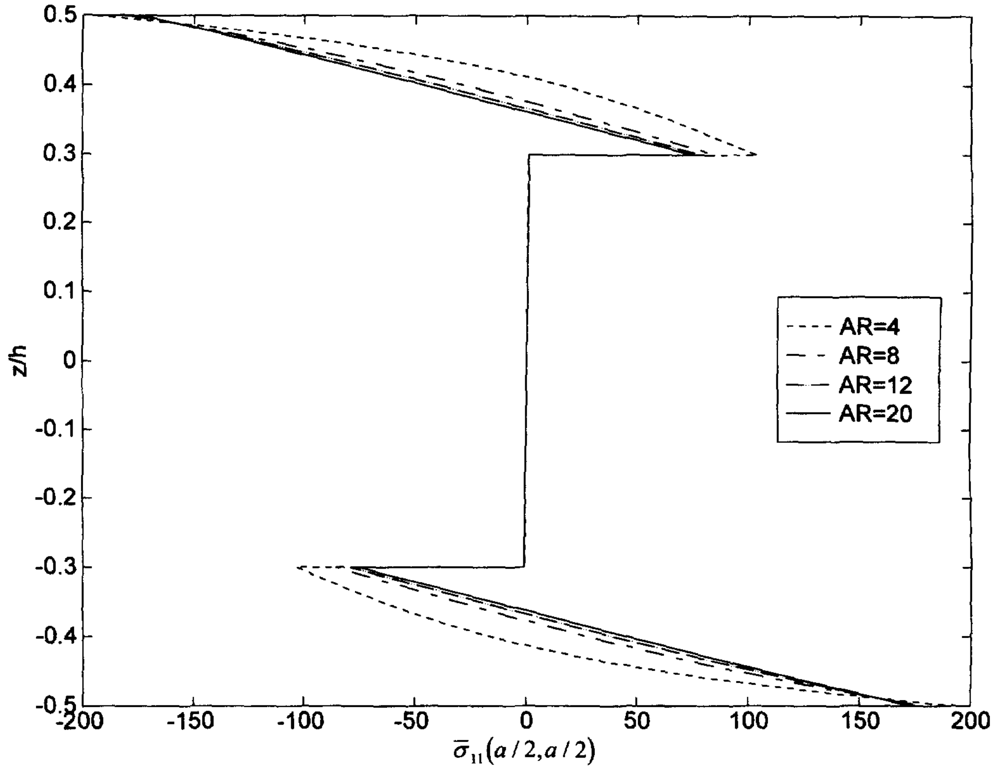


(b)

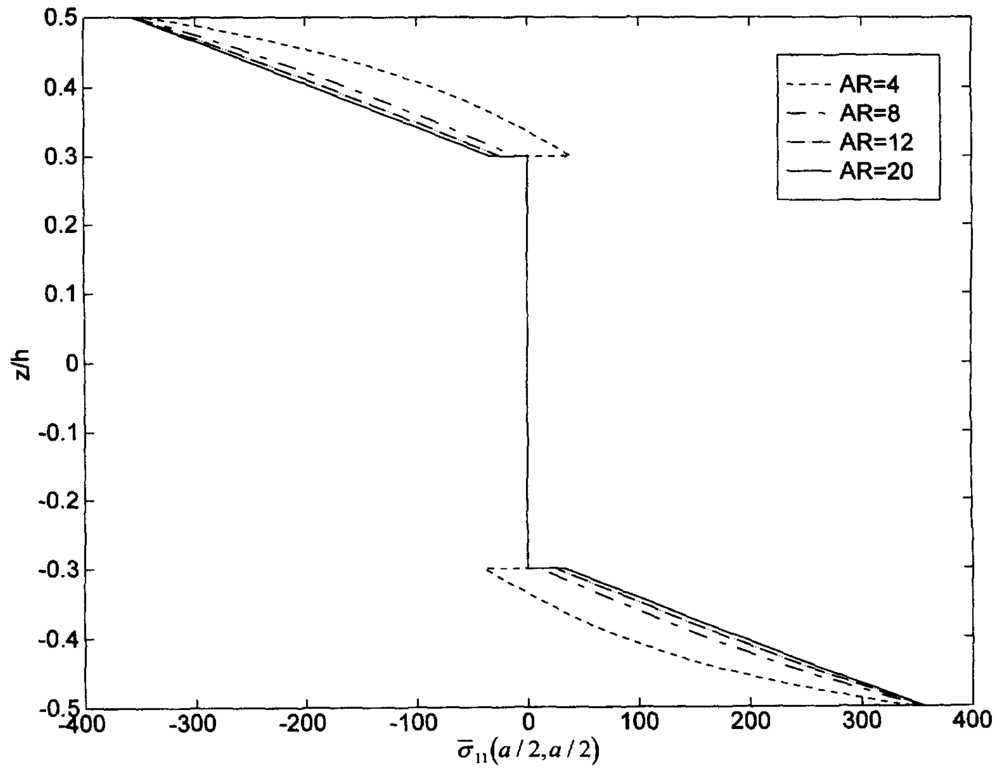


(b′)

Fig. 2—*Continued.*

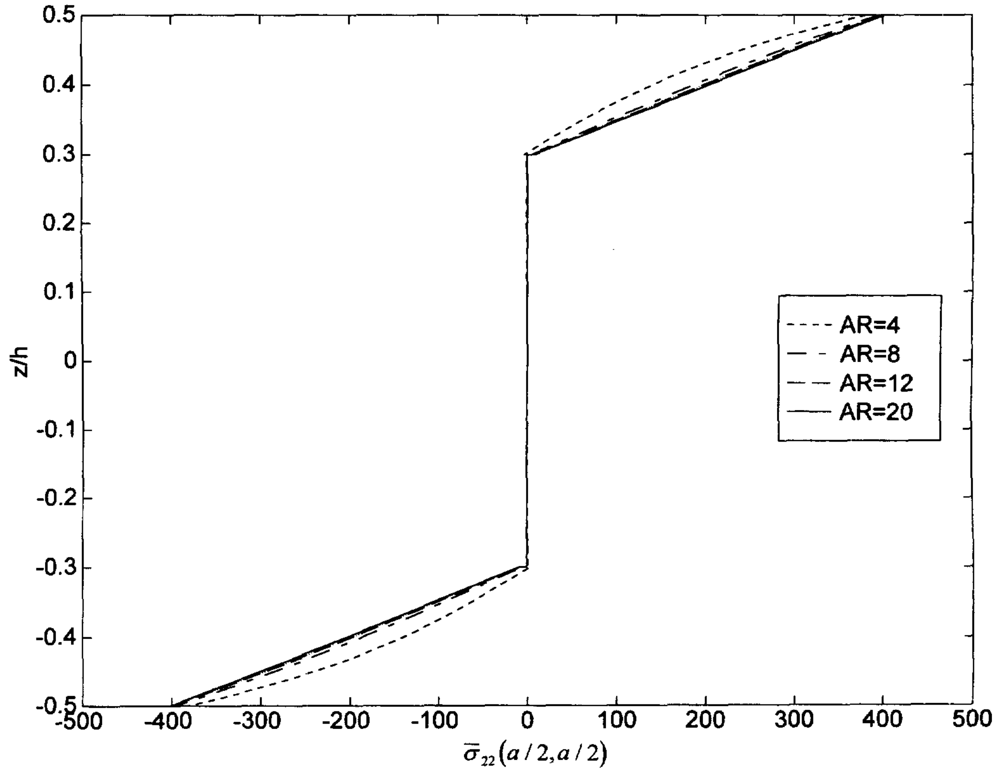


(c)

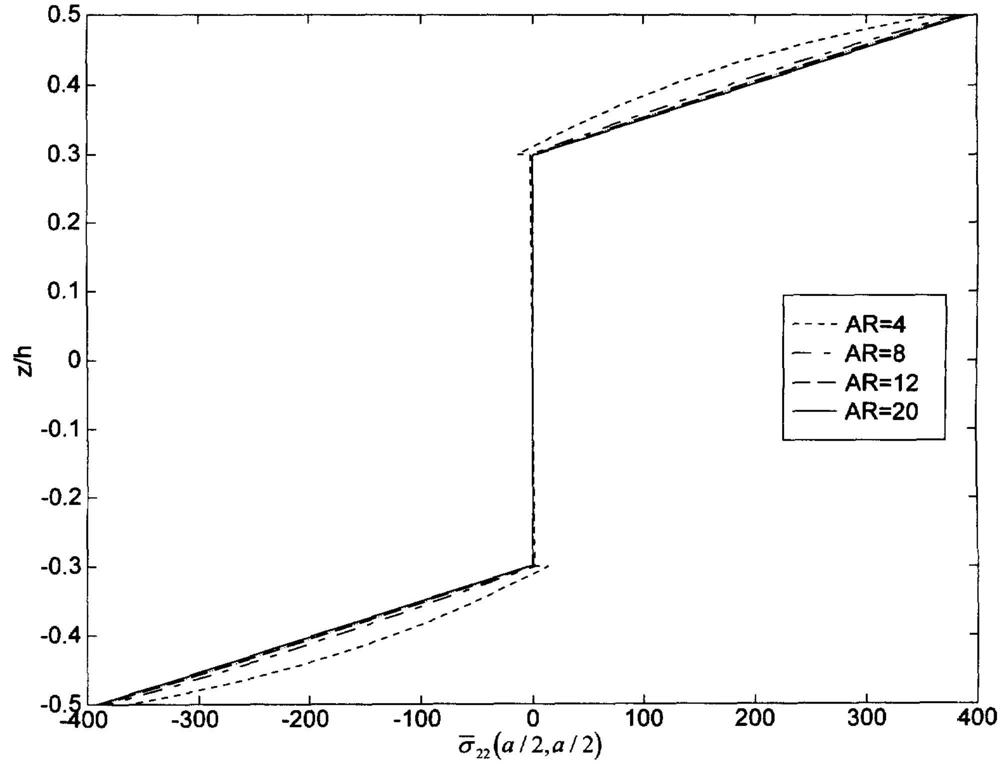


(c′)

Fig. 2—*Continued.*

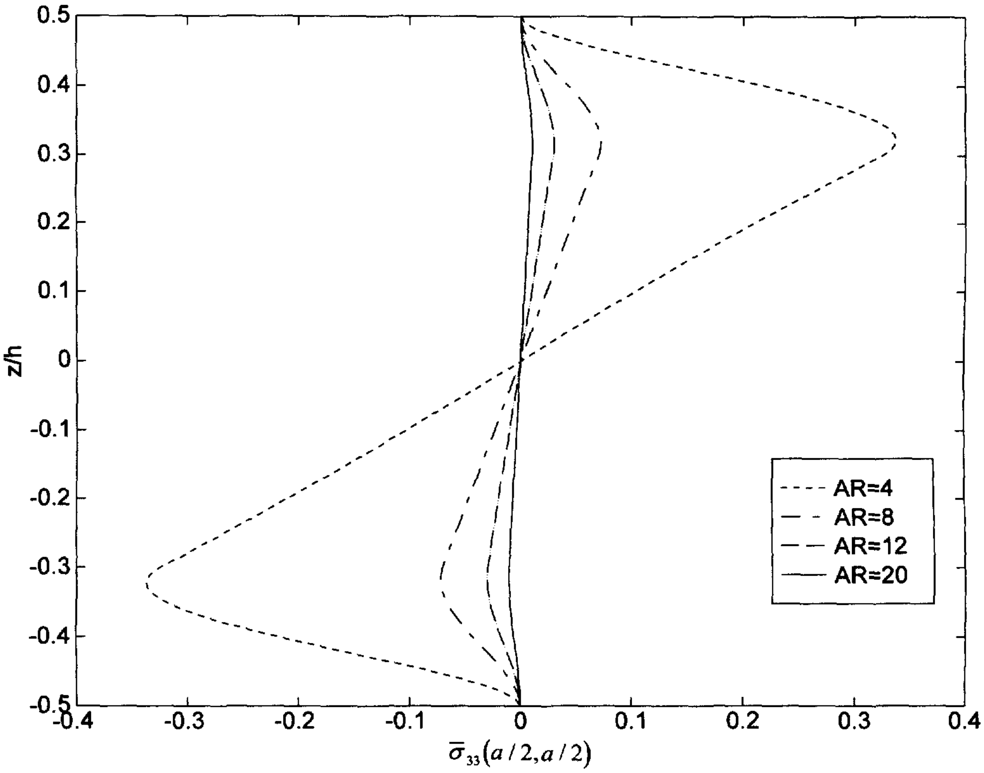


(d)

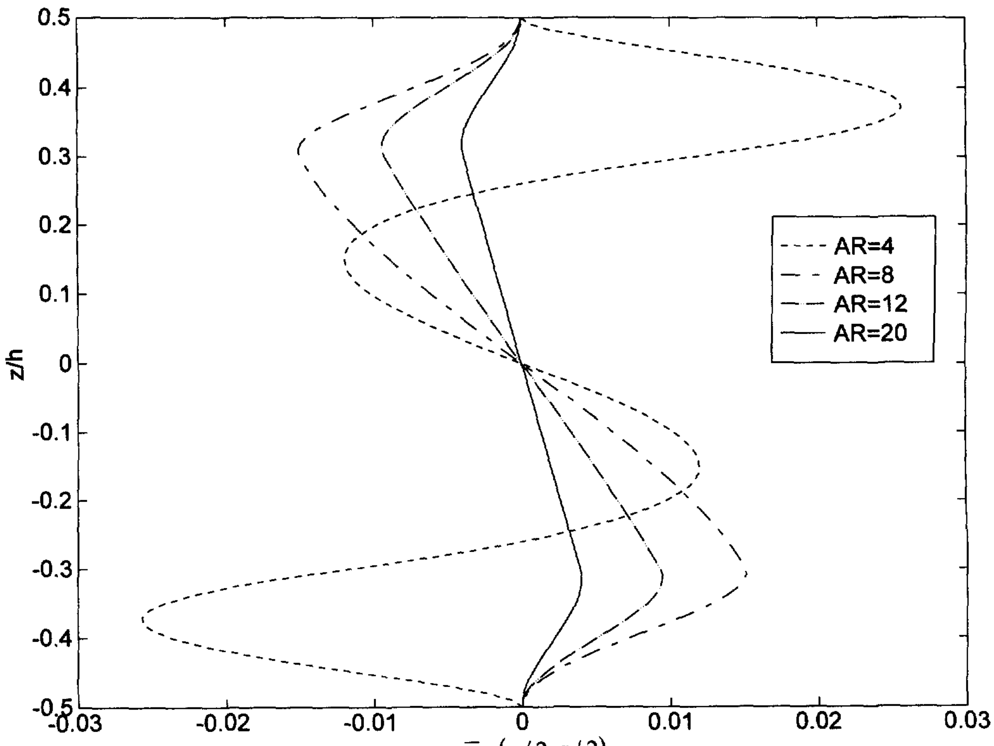


(d′)

Fig. 2—*Continued.*

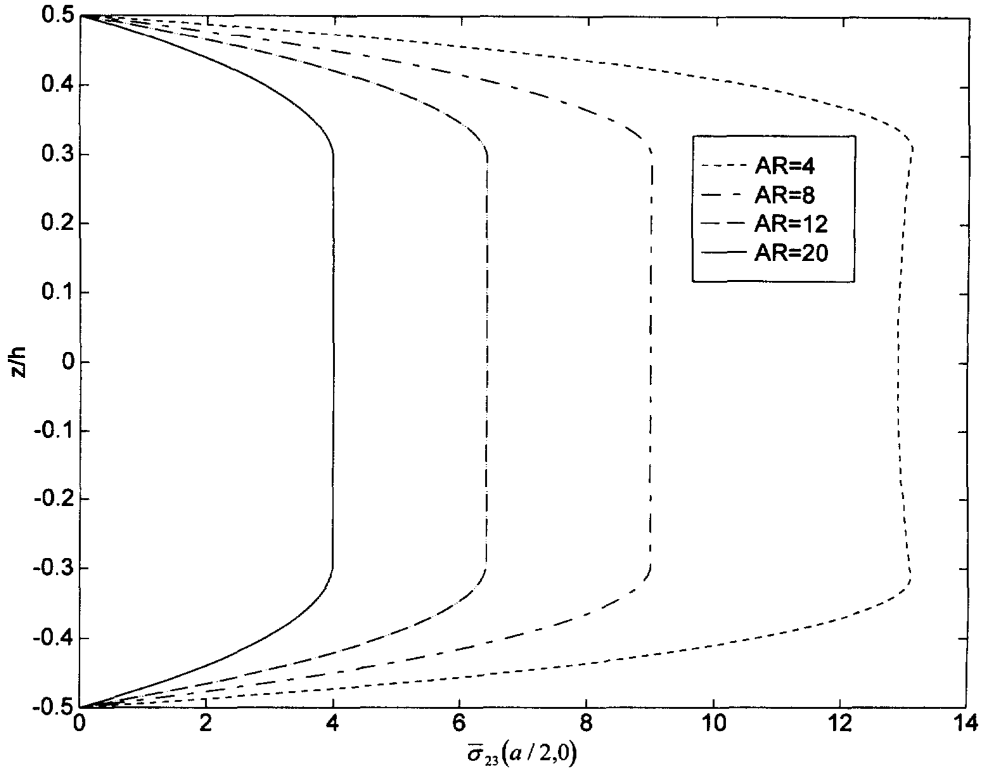


(e)

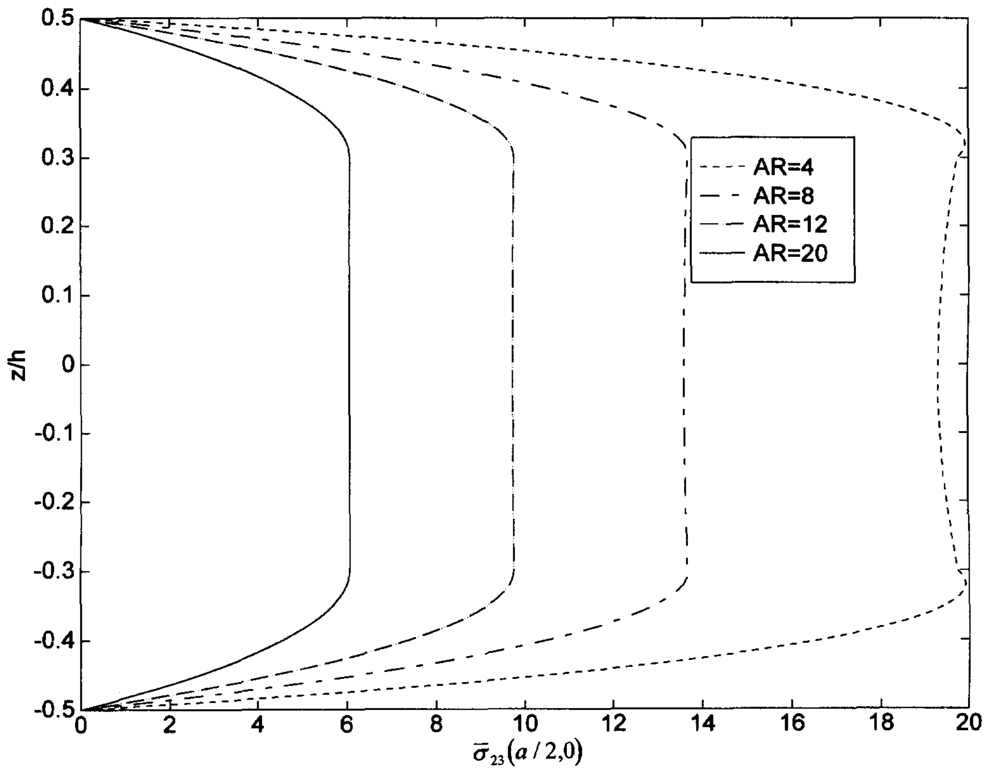


(e′)

Fig. 2—*Continued.*

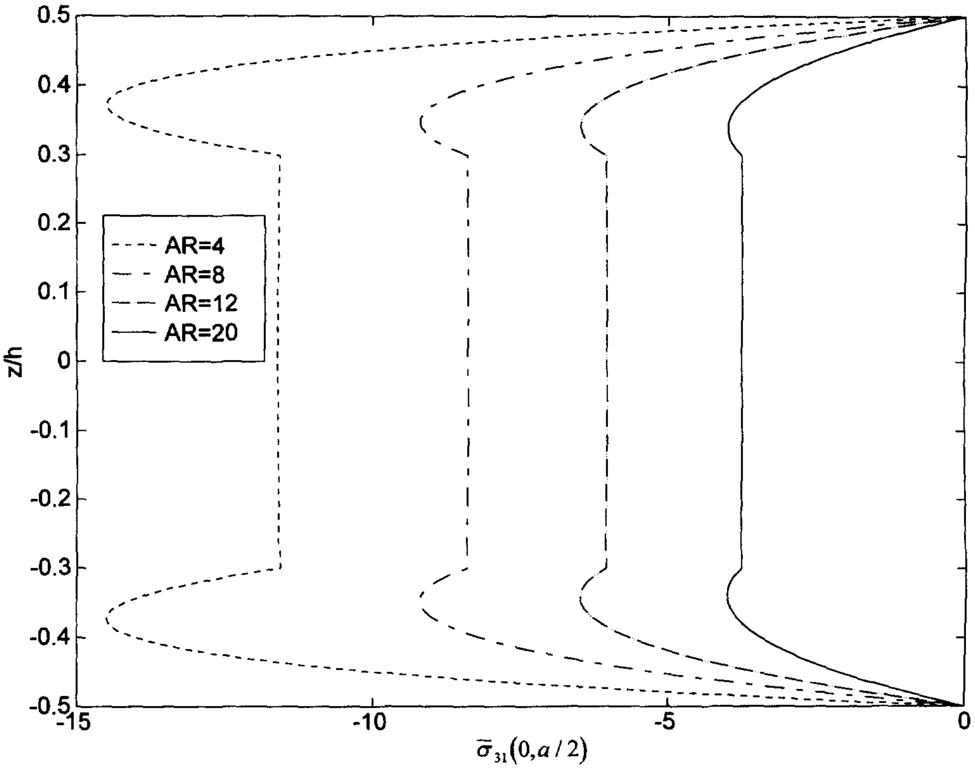


(f)

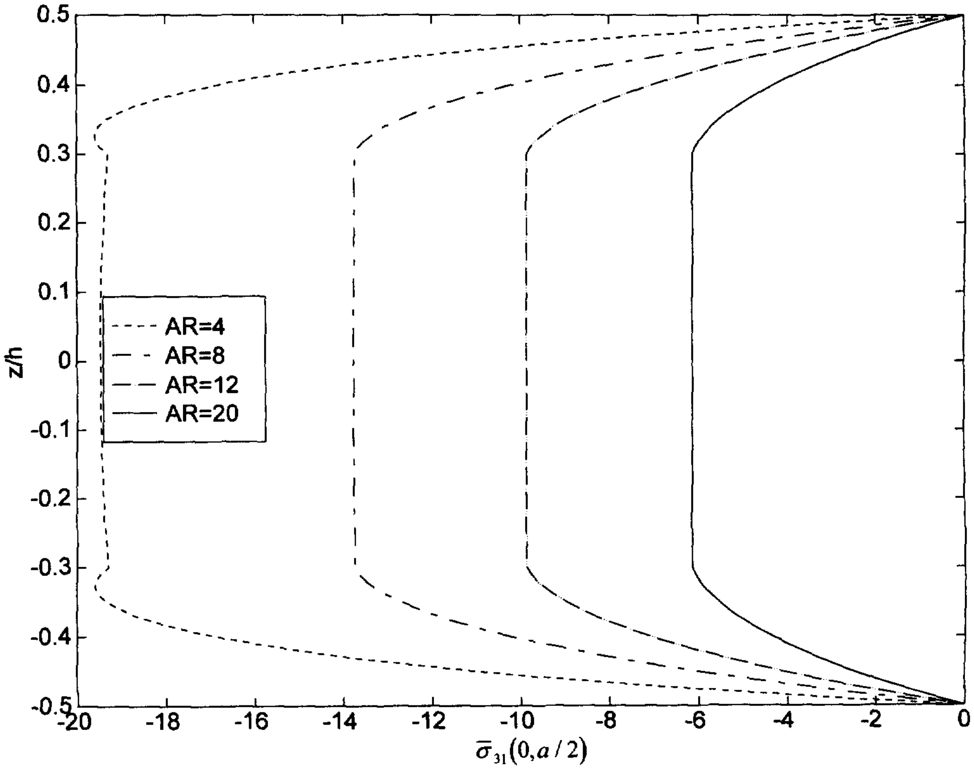


(f′)

Fig. 2—*Continued.*

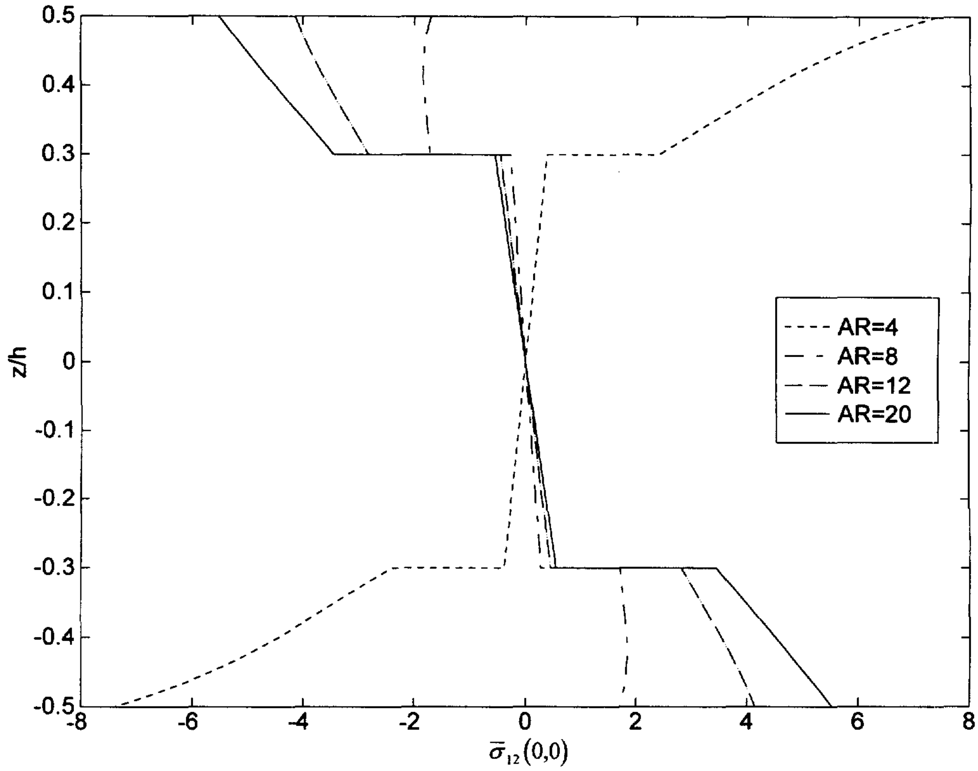


(g)

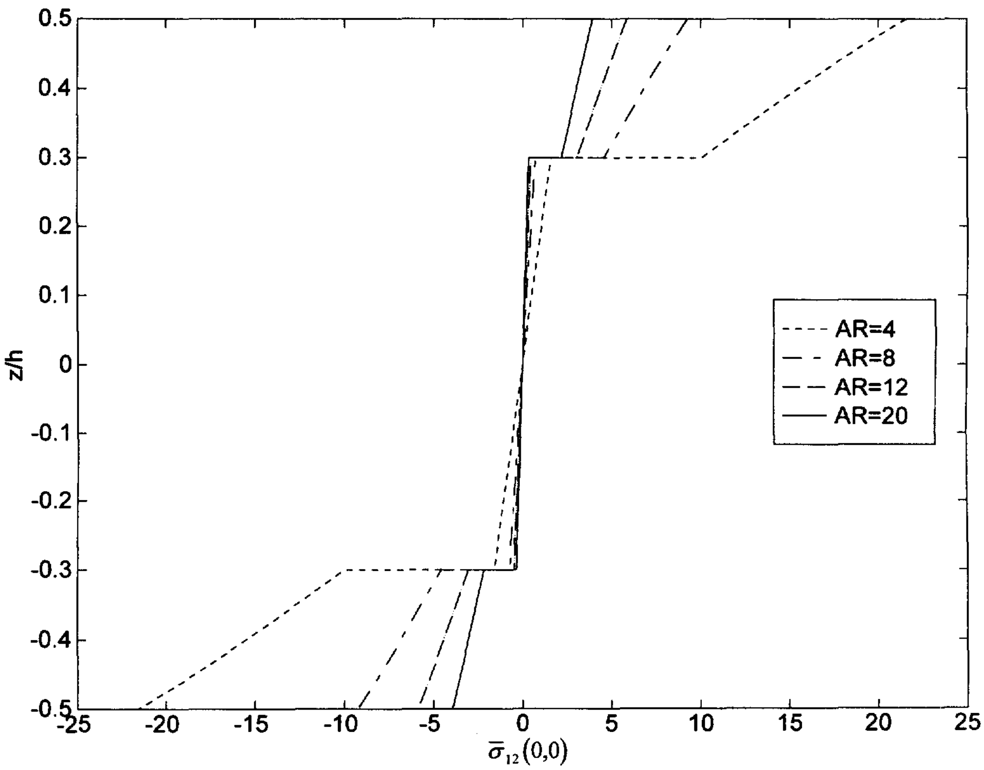


(g′)

Fig. 2—*Continued.*

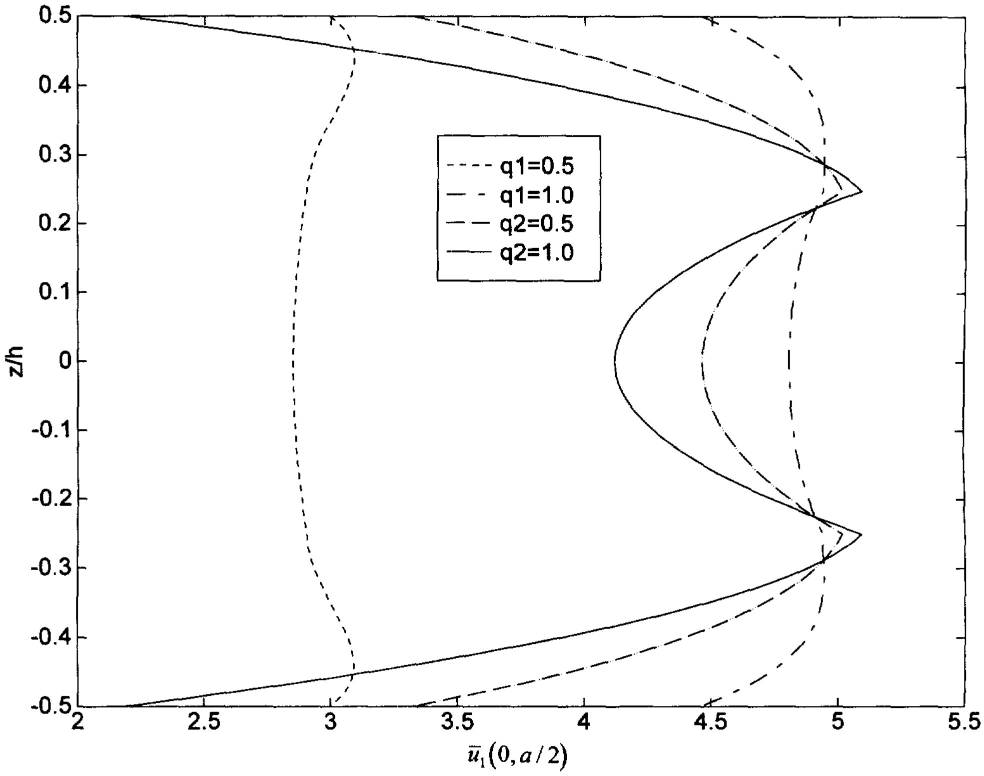


(h)

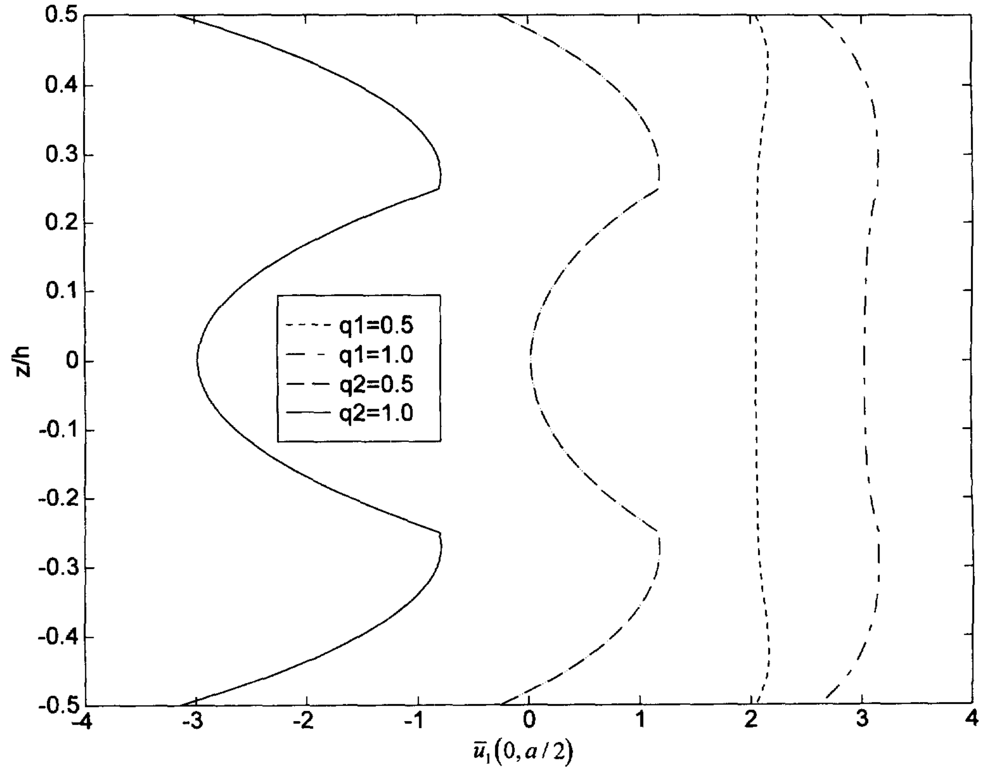


(h′)

Fig. 2—*Continued.*

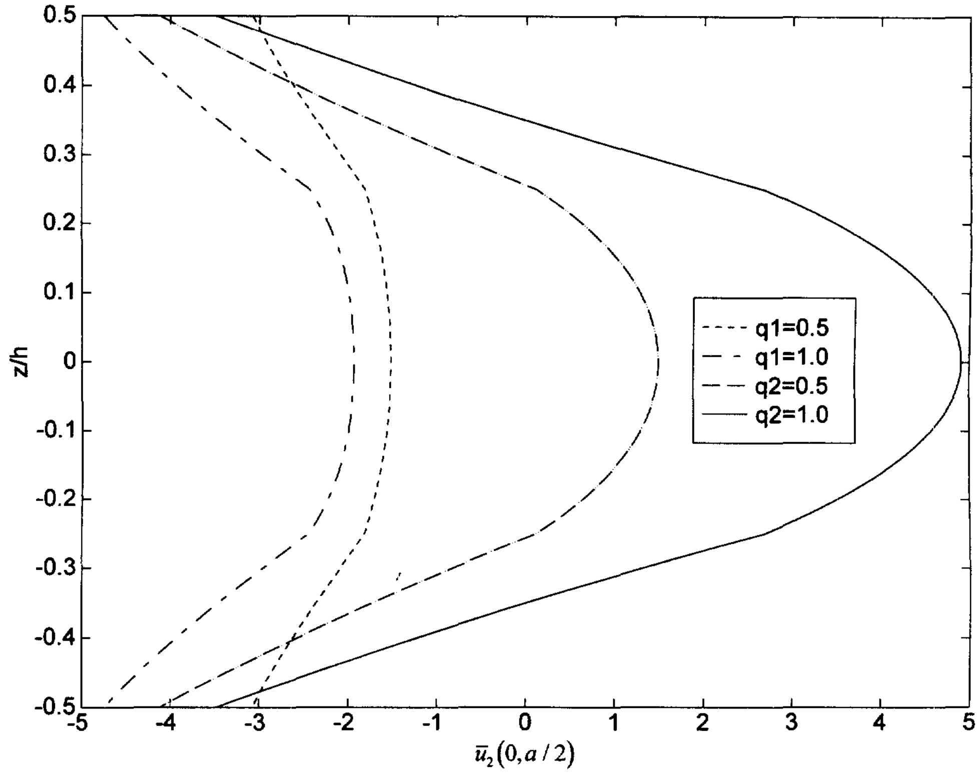


(a)

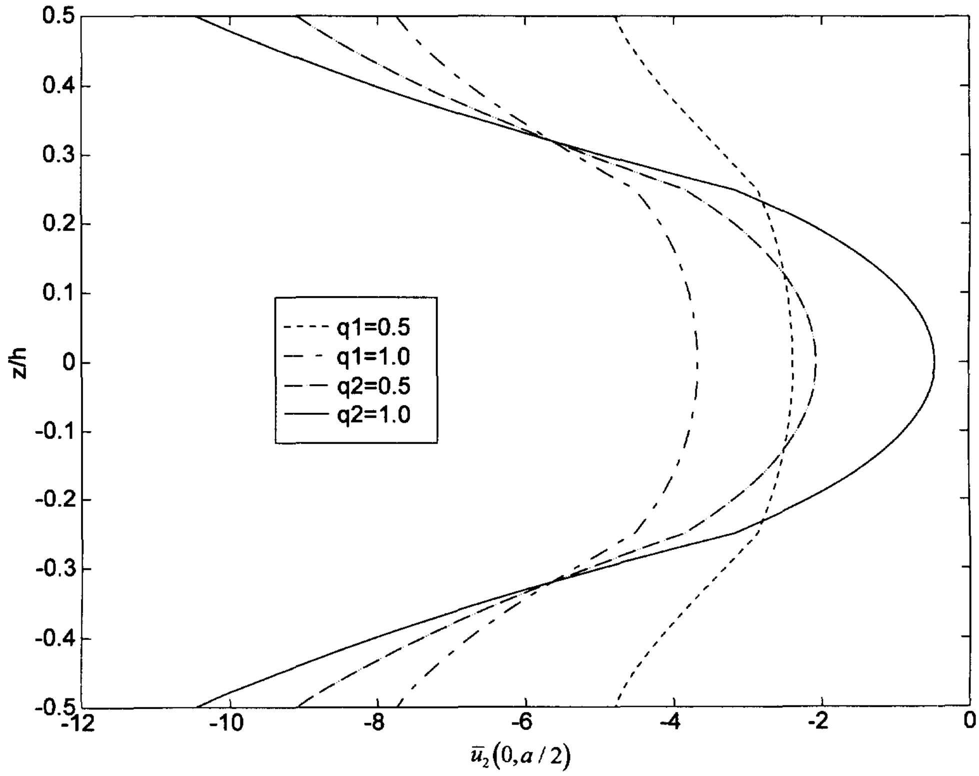


(a′)

Fig. 3. Transient analysis of a cross-ply laminate (0°, 90°, 90°, 0°), with material I for all the layers ($h = 2$, $a/h = 8$), and suddenly exposed to a temperature change $\bar{T}$ (at the time $t = 0$), uniform over the plate surfaces. The first term only in the temperature expansion is considered ($m_1 = m_2 = 1$). Non-dimensional in-plane displacements (a, a', b, b') $\bar{u}_1$ and $\bar{u}_2$; Non-dimensional stresses (c, c', d, d', e, e', f, f', g, g', h, h') $\bar{\sigma}_{11}$ $\bar{\sigma}_{22}$ $\bar{\sigma}_{33}$ $\bar{\sigma}_{23}$ $\bar{\sigma}_{31}$ $\bar{\sigma}_{12}$

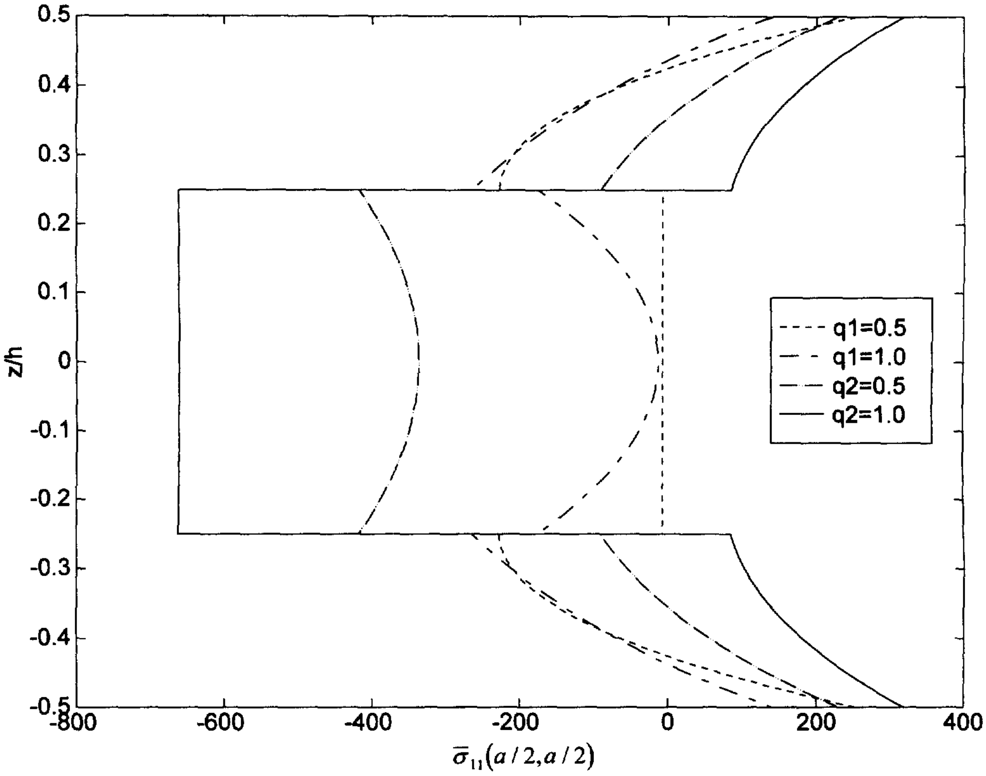


(b)

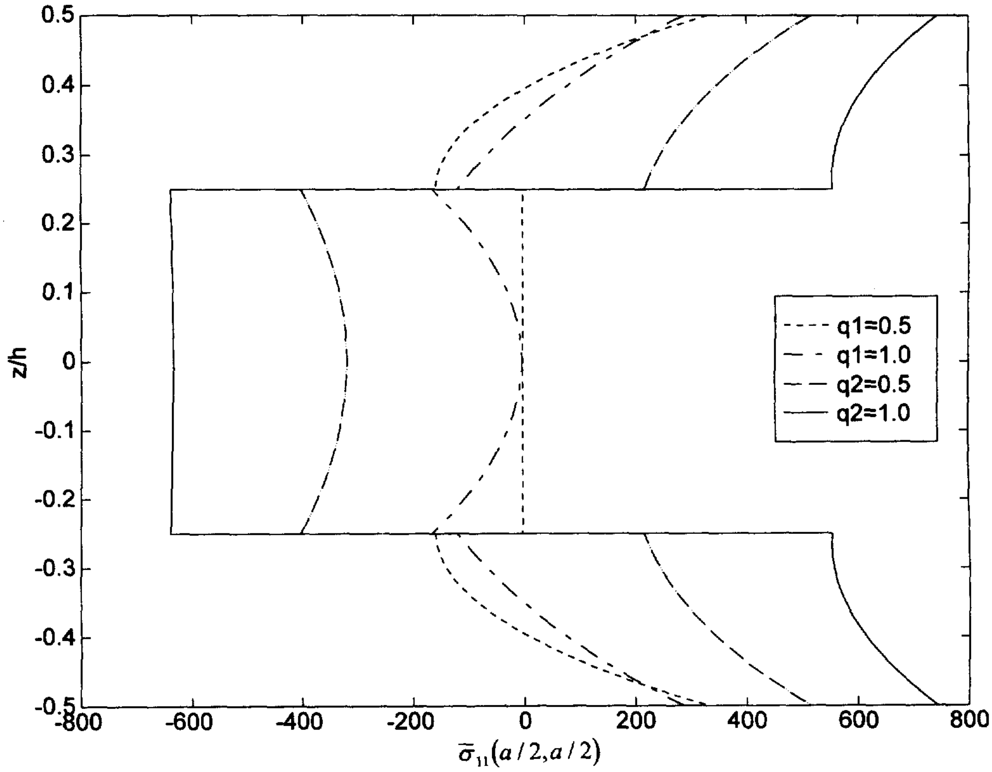


(b′)

Fig. 3—*Continued.*

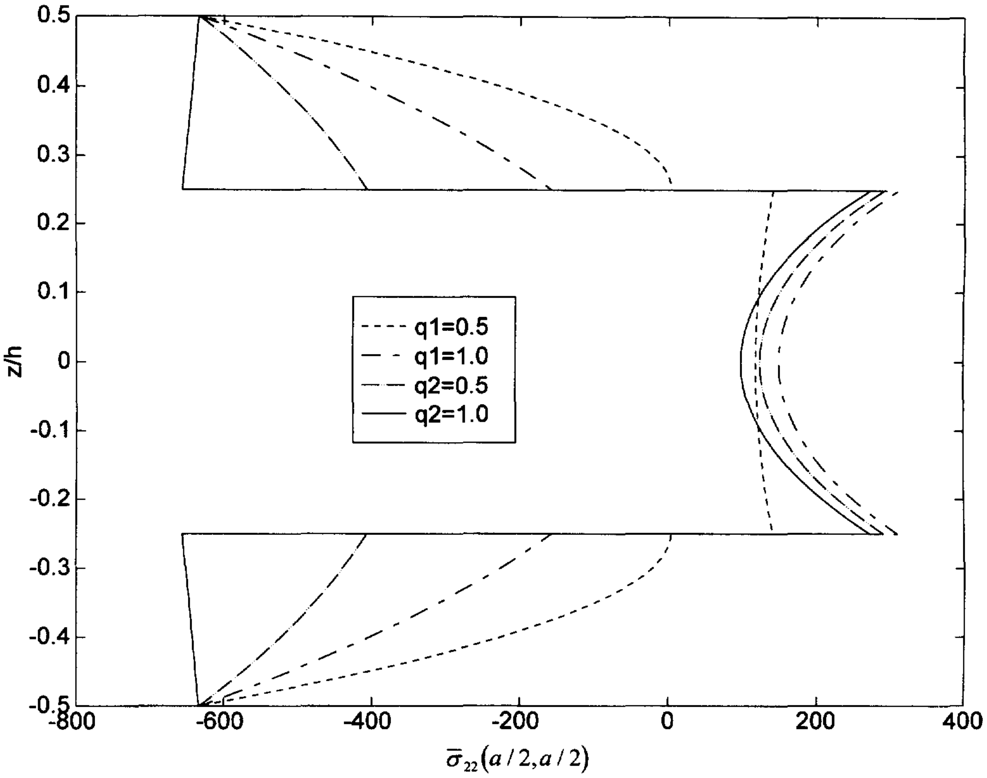


(c)

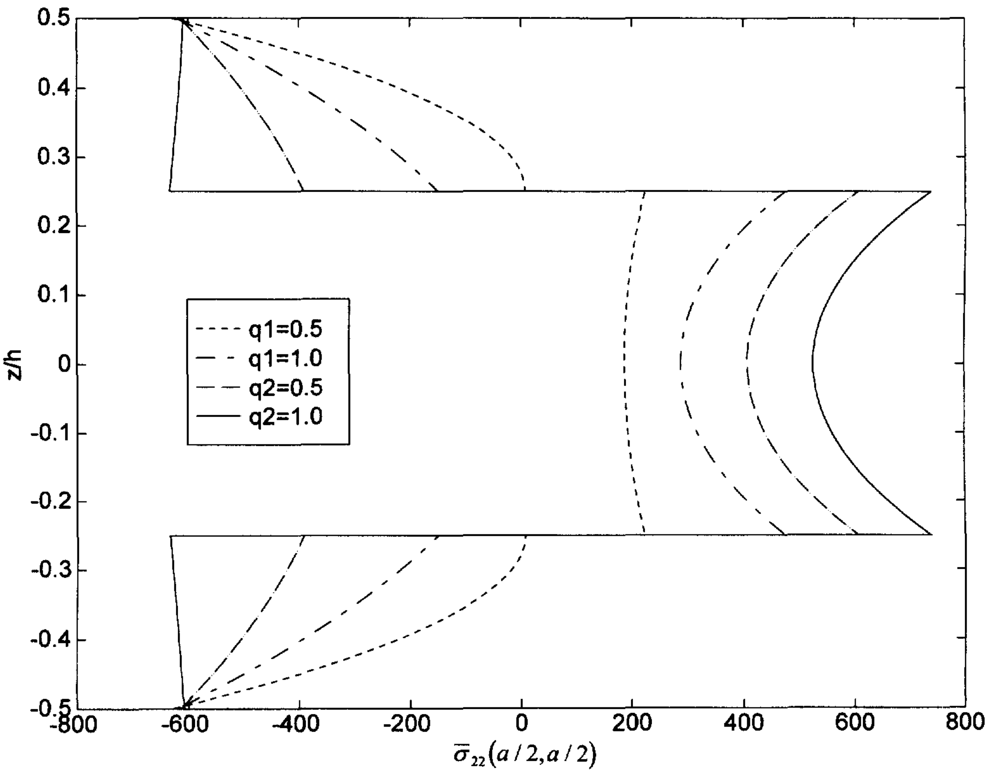


(c′)

Fig. 3—*Continued.*

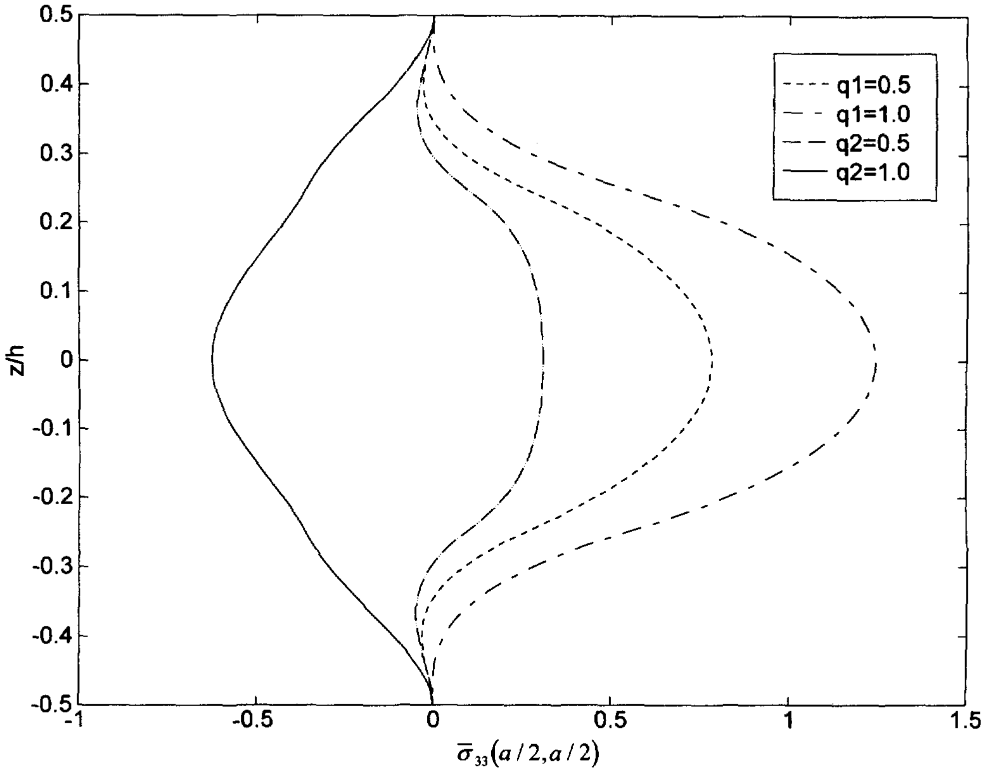


(d)

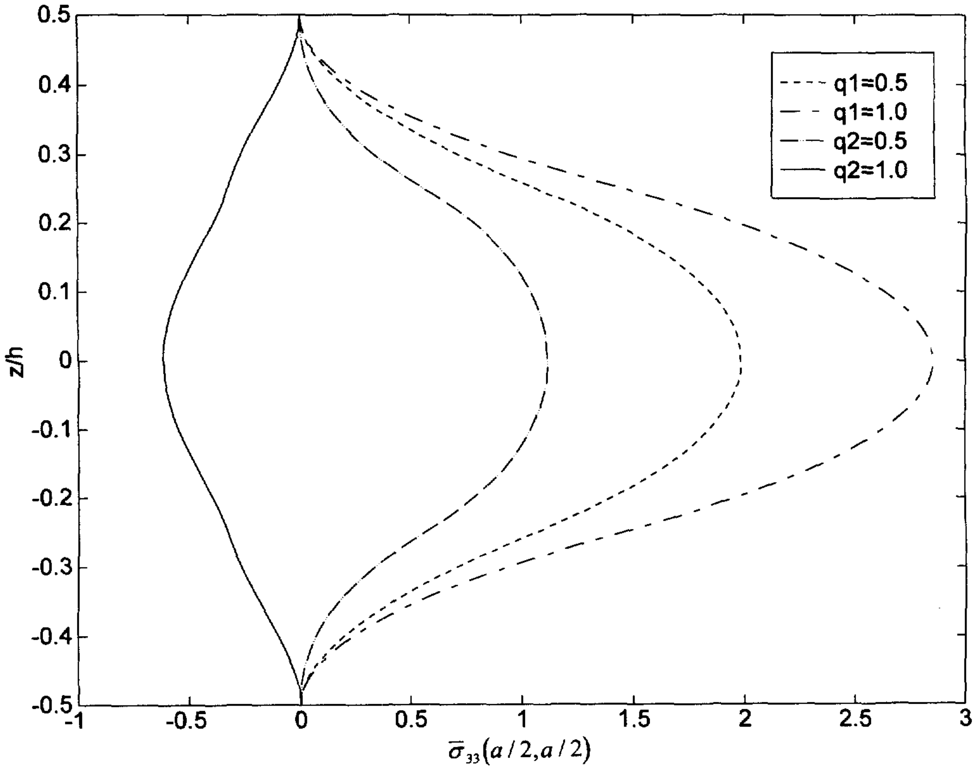


(d′)

Fig. 3—*Continued.*

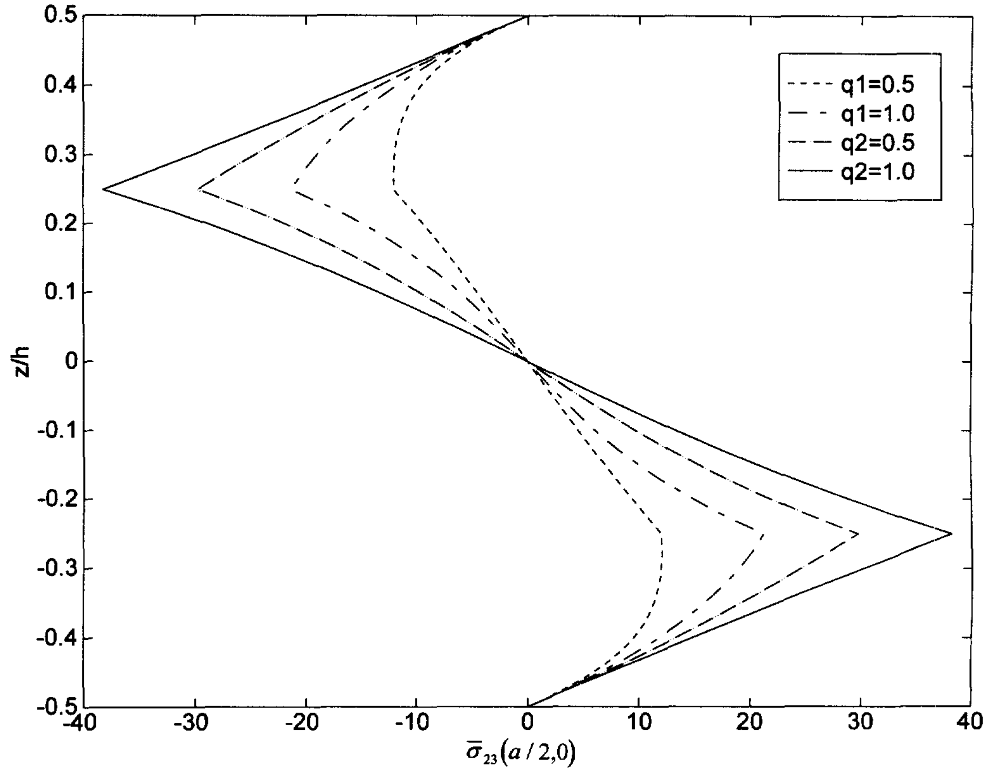


(e)

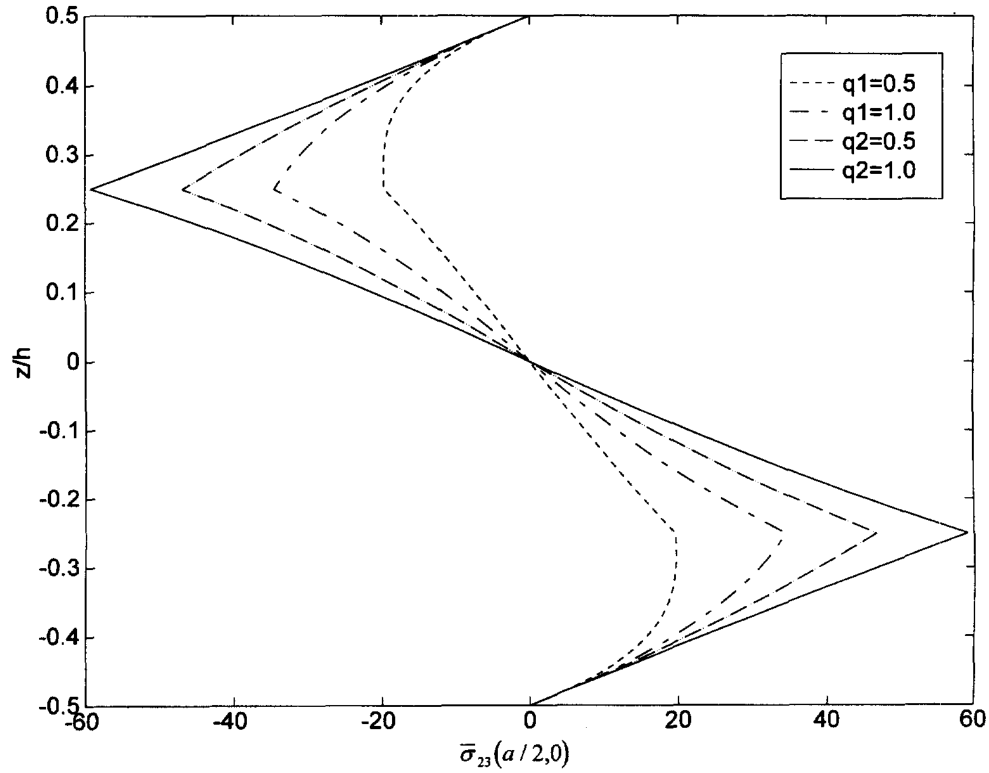


(e′)

Fig. 3—*Continued.*

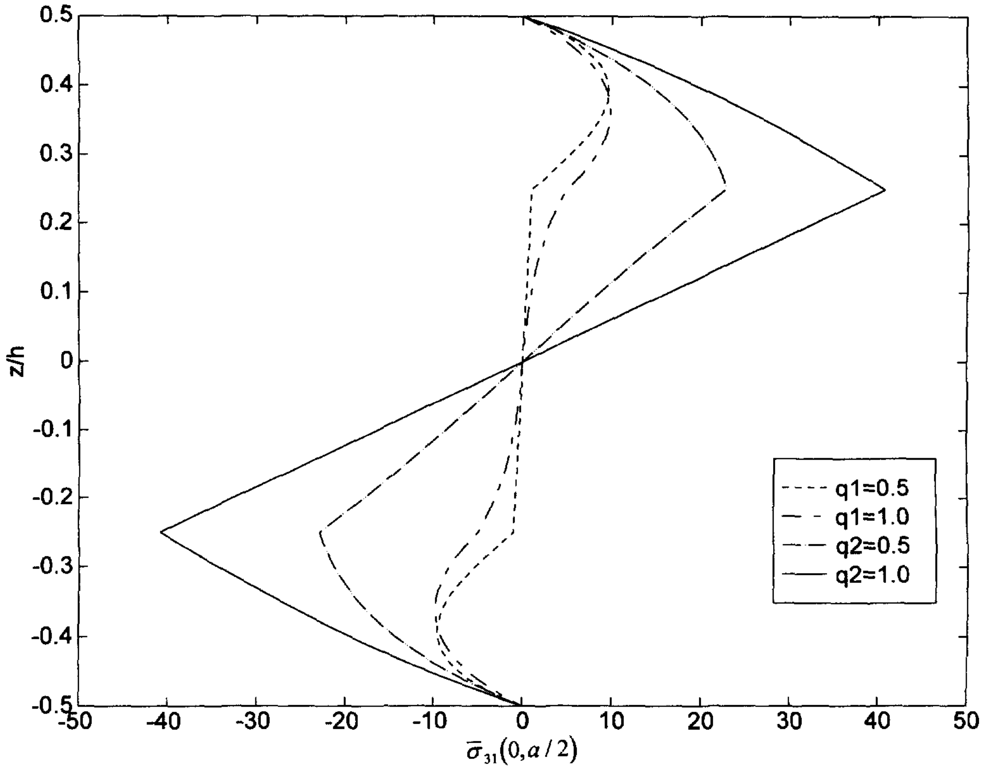


(f)

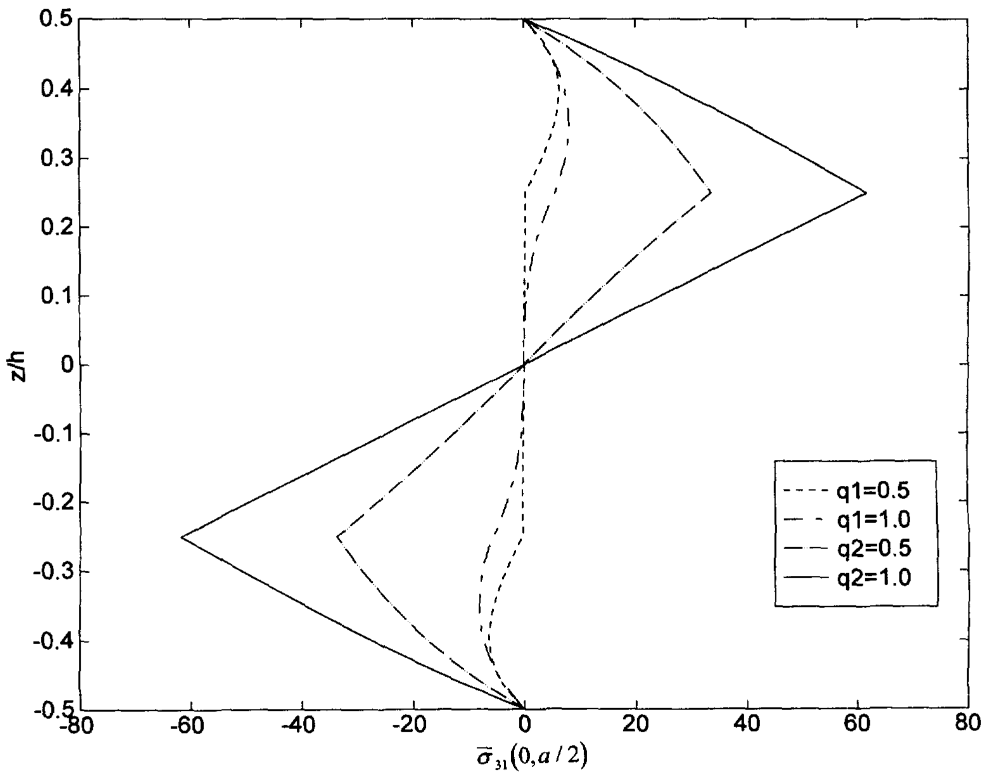


(f′)

Fig. 3—*Continued.*

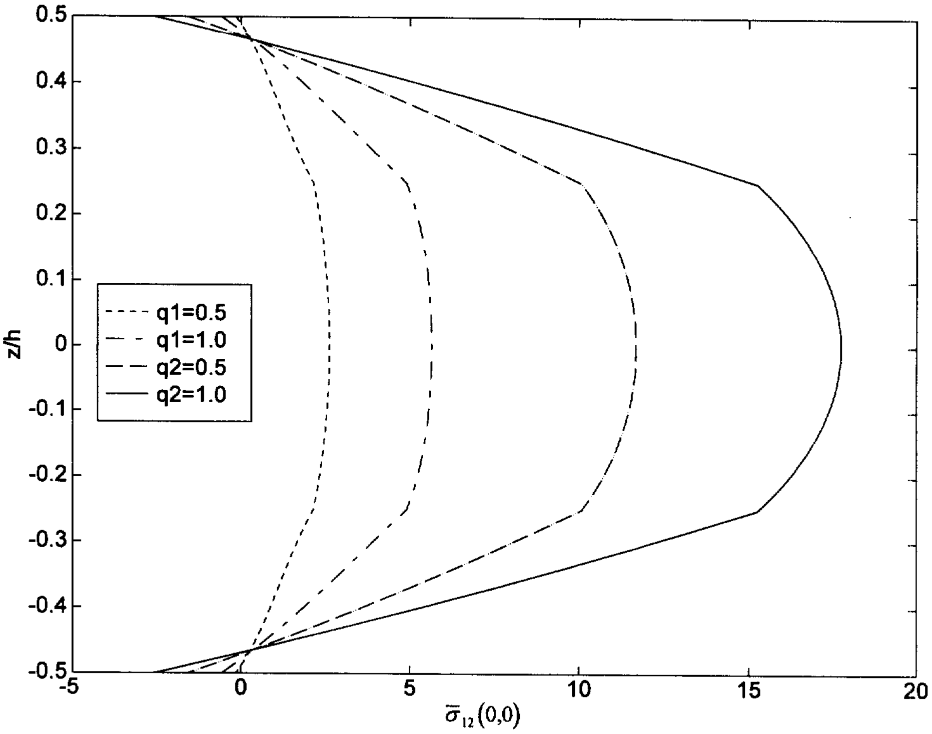


(g)

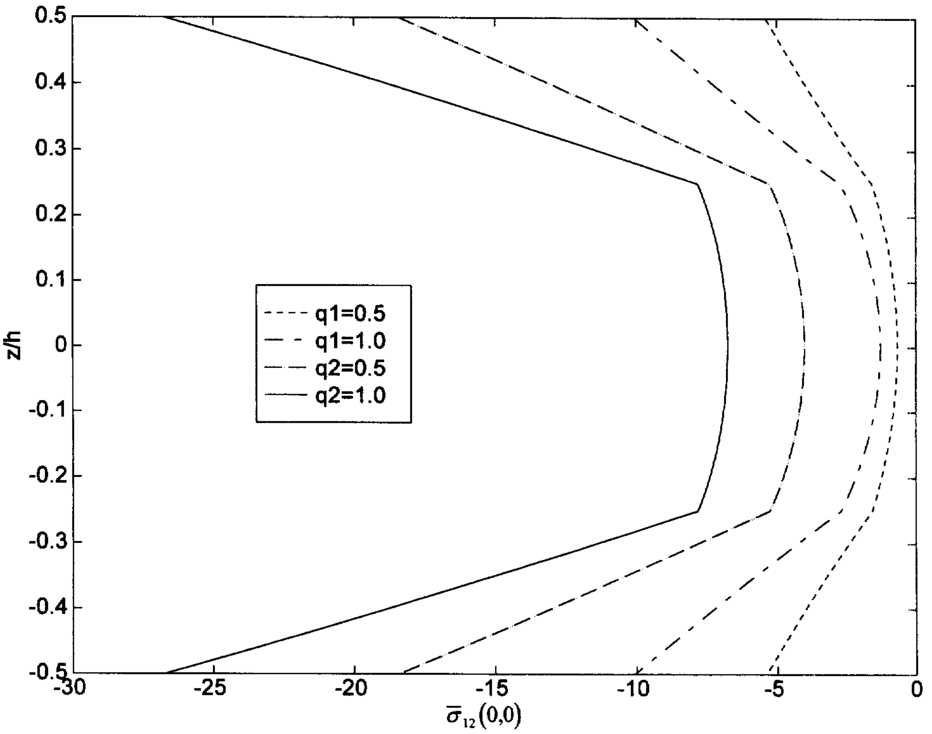


(g′)

Fig. 3—*Continued.*



(h)



(h′)

Fig. 3—*Continued.*

Nondimensionalization

$$\bar{\sigma}_{ij} = \frac{\sigma_{ij}}{\alpha_r \bar{T} E_r}, \quad \bar{u}_i = \frac{u_i}{\alpha_r \bar{T} h}(\alpha_r = 10^{-6}/^{\circ}\mathrm{K}, \quad E_r = 1\,\mathrm{Gpa}).$$

The original erroneous results in Savoia and Reddy (1995) are depicted by unprimed plotting in Figs 2 and 3, whereas the corrected results are shown by primed plottings. The qualitative behaviors of original results are quite different from those of corrected results as shown in Figs 2 and 3.

As shown in the Fig. 2(h), it was reported in the Savoia and Reddy (1995) that the in-plane shear stress $\bar{\sigma}_{12}$ can be completely reversed as the thickness changed. However, corrected solution of in-plane shear stress $\bar{\sigma}_{12}$ in Fig. 2(h′) is not in accordance with above reported results. Even though it was mentioned in Savoia and Reddy (1995) that transverse normal stress is small compared with the other stress components, its smallness in the order of magnitude should be emphasized. Also in thick case ($h/L = 4$), the higher oscillatory behavior of transverse normal stress than that mentioned in the original paper (Savoia and Reddy (1995)) are observed as demonstrated in the Fig. 2(e) and (e′). In addition, as the thickness ratio of the plates gets smaller, the sign of the transverse normal stresses are opposite to the predicted results of Savoia and Reddy (1995).

In Fig. 3, the transient time responses are depicted. The corrected in-plane normal stress of the transient time response behavior shows totally different behavior from those of original response. Especially transition to steady states of two results in Fig. 3(h) and (h′) shows qualitative discrepancy.

The corrected results presented here should serve as benchmark solutions when the approximate models are developed so as to analyze the thermoelastic response of the composite laminates.

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